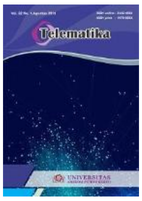

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# Portfolio Risk Assessment Using VaR and CVaR: A Comparative Study of Variance–Covariance Method and Monte Carlo Simulation

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**ABSTRACT**

This study examines portfolio risk in Indonesia's energy sector by applying Value at Risk (VaR) and Conditional Value at Risk (CVaR) under the Variance–Covariance and Monte Carlo Simulation approaches. The analysis focuses on ten stocks from the oil and gas as well as coal subsectors listed on the Indonesia Stock Exchange (IDX), using monthly closing price data from January 2020 to December 2024. A Weighted Scoring Method (WSM) is first employed to select stocks with superior fundamentals and liquidity, based on market capitalization, return on equity, debt-to-equity ratio, net profit margin, trading volume, and dividend yield. An optimal portfolio is then constructed using the Maximum Sharpe Ratio (MSR) framework, resulting in a portfolio dominated by PTBA, MEDC, and MBAP. Portfolio risk is subsequently estimated using VaR and CVaR at the 95% and 99% confidence levels under both the Variance–Covariance and Monte Carlo approaches. The empirical results indicate that CVaR consistently produces higher risk estimates than VaR, highlighting its superior ability to capture tail risk. Furthermore, the Variance–Covariance method yields slightly more conservative CVaR estimates compared to Monte Carlo Simulation, which is attributed to the near-normal distribution of portfolio returns during the observation period. Model validity is confirmed through backtesting using the Kupiec test, which shows that the VaR estimates satisfy statistical adequacy criteria. Overall, the findings suggest that while the Variance–Covariance approach remains effective under normality assumptions, Monte Carlo Simulation offers greater flexibility in modeling extreme market conditions. This study contributes to the literature by providing empirical evidence on comparative risk estimation methods in Indonesia's highly volatile energy sector.

## 1. INTRODUCTION

The capital market plays a crucial role in the Indonesian economy by serving as a mechanism for capital mobilization, supporting business development, and encouraging public participation in investment activities. In investment decision-making, investors are consistently confronted with two fundamental aspects, namely return and risk (Fabozzi & Peterson, 2003). To manage risk effectively, investors need to construct a financial risk management portfolio, which constitutes a systematic process of identifying, measuring, evaluating, and controlling risk within a set of investment assets in order to achieve financial objectives while balancing expected returns and acceptable risk levels.

In the practice of portfolio risk management, Value at Risk (VaR) has long been employed as a market risk measure capable of estimating the maximum potential loss at a given confidence level (Jorion, 2007). Numerous studies have focused on improving VaR estimation through the development of volatility models, such as GARCH and range-based volatility models (Padmakumari & Shaik, 2023; Farikha et al., 2024; Darmanto et al., 2025). Nevertheless, VaR suffers from a fundamental limitation, namely its inability to capture losses beyond the VaR threshold (tail losses), rendering it less effective in measuring extreme risk, particularly under turbulent market conditions (Takaishi, 2023; Najamuddin et al., 2024).

To address the shortcomings of VaR, Conditional Value at Risk (CVaR), also known as Expected Shortfall, was developed as a coherent and more informative risk measure. CVaR estimates the expected loss conditional on losses exceeding the VaR threshold, thereby providing a more accurate representation of extreme risk (Rockafellar & Uryasev, 2000; Kithinji et al., 2021). CVaR has been widely applied in various contexts, including portfolio optimization (Rockafellar & Uryasev, 2000; Prihatiningsih et al., 2020), derivative pricing (Kosapong et al., 2025), and extreme volatility-based risk assessment (Widiastuti et al., 2025).

The Variance–Covariance method has been extensively used in estimating VaR and CVaR due to its mathematical simplicity and ease of implementation, particularly when the assumptions of normally distributed returns and stable volatility are satisfied (Jorion, 2007; Sarykalin et al., 2014). Under relatively stable market conditions, this parametric approach often provides consistent and reliable risk estimates.

However, empirical evidence suggests that the normality assumption underlying the Variance–Covariance method is frequently violated, especially in markets characterized by high volatility and fat-tailed return distributions (Hong et al., 2014; Najamuddin et al., 2024). As an alternative, Monte Carlo Simulation offers greater flexibility by allowing the simulation of various return distributions, dependency structures among assets, and extreme market scenarios that are difficult to capture using the Variance–Covariance approach (Rubinstein, 1981; Khan et al., 2024).

Previous studies indicate that Monte Carlo Simulation provides superior flexibility by accommodating non-normal return behavior, fat tails, and complex inter-asset dependencies (Vaniya et al., 2022; Hong et al., 2014). Moreover, Monte Carlo-based methods are widely employed to enhance the accuracy of VaR and CVaR estimation, particularly under volatile and nonlinear market conditions (Indarwati & Kusumawati, 2021; Hong et al., 2014).

Despite the extensive application of Value at Risk (VaR) and Conditional Value at Risk (CVaR) in financial risk management, several gaps remain in the existing literature. Most empirical studies focus on a single estimation approach, either the parametric Variance–Covariance method or simulation-based techniques, without conducting a systematic comparative analysis under identical portfolio structures and market conditions. Moreover, comparative studies are largely concentrated in developed markets, while empirical evidence from emerging markets, particularly Indonesia, remains limited despite their distinct characteristics of higher volatility and fat-tailed return distributions. These limitations highlight the need for a comprehensive comparative analysis of Variance–Covariance and Monte Carlo Simulation methods in estimating tail risk in the Indonesian capital market.

The energy sector in Indonesia, particularly the oil, gas, and coal subsectors, plays a strategic role in the national economy and contributes significantly to the capital market. Nevertheless, this sector is also known for its relatively high volatility. Furthermore, energy stocks generally exhibit strong liquidity and are supported by solid corporate fundamentals, making them a relevant and representative object for quantitative risk-based analysis.

Based on the above background, a comparative analysis between the Variance–Covariance method and Monte Carlo Simulation becomes essential to evaluate their respective performance in estimating VaR and CVaR. This analysis is applied to energy sector stocks in Indonesia, which are characterized by high volatility. The comparative approach is expected to provide a more comprehensive understanding of the most appropriate risk measurement methods in accordance with the volatility characteristics and market dynamics of Indonesia's energy sector.

## 2. RESEARCH METHODS

### 2.1. Maximum Sharpe Ratio Portfolio

A stock portfolio is a combination of selected stocks constructed based on specific criteria with the objective of achieving an optimal level of return while minimizing potential risk (Fabozzi & Peterson, 2003). Suppose an investor allocates capital across N stocks with a return vector $r = (r_i, r_2 \cdots, r_N)'$ an expected return vector $\mu$ and a covariance matrix $\Sigma$. Given a portfolio weight vector $w$, the expected portfolio return is expressed as $\mu_p = w'\mu$ while portfolio risk is measured by the portfolio variance $\sigma_p^2 = w'\Sigma w$.

The Maximum Sharpe Ratio (MSR) portfolio is a widely used optimal portfolio model in investment decision-making, as it effectively balances the trade-off between return and risk. The MSR portfolio is obtained by solving an optimization problem that maximizes the Sharpe ratio, defined as the ratio of portfolio excess return to portfolio risk, subject to relevant investment constraints such as full investment and other allocation restrictions.

Let $R_f$ denote the risk-free rate of return, which represents the return on a riskless asset over the same investment horizon, the Maximum Sharpe Ratio (MSR) portfolio is obtained by solving the following optimization problem:

$$\max \; \frac{w^T\mu - R_f}{\sqrt{w^T \sum w}} \qquad (1)$$
$$\text{constraint}: \; 1^T w = 1, \qquad w \geq 0$$

### 2.2. Value at Risk (VaR)

Value at Risk (VaR) is one of the most extensively employed risk measures in financial risk management. It quantifies the maximum potential loss of a portfolio that is not expected to be exceeded over a specified time horizon at a given confidence level, under normal market conditions (Jorion, 2007). As such, VaR provides a probabilistic assessment of downside risk and has become a standard tool for regulatory and internal risk control purposes.

Formally, for a confidence level $(1-\alpha)$ and a holding period $t$, VaR is defined as the $\alpha$-quantile of the loss distribution of portfolio returns and can be expressed as:

$$VaR_\alpha = W_0 \, R_\alpha{}^* \sqrt{t} \qquad (2)$$

where $W_0$ denotes the initial investment value, and $R_\alpha{}^*$ represents the $\alpha$-quantile of the portfolio return distribution.

Despite its widespread adoption, VaR suffers from several well-documented limitations. Most notably, VaR does not provide any information regarding the magnitude of losses beyond the VaR threshold (tail losses). Consequently, it may underestimate risk exposure during periods of extreme market movements or heightened volatility, when the distribution of returns deviates from normality (Takaishi, 2023).

#### 2.3.1. VaR Estimation Using the Variance–Covariance Method

One of the most commonly used parametric approaches for estimating VaR is the Variance–Covariance (VC) method, which assumes that asset returns follow a multivariate normal distribution and that portfolio returns can be fully characterized by their mean and variance (Jorion, 2007).

Under this framework, VaR is computed as:

$$VaR_{VC} = Z_\alpha \, \sigma_p \sqrt{t} \, \mathrm{W}_0 \qquad (3)$$

where $Z_\alpha$denotes the critical value from the standard normal distribution corresponding to confidence level α, and $\sigma_p$ is the portfolio standard deviation, calculated as $\sqrt{w^T \sum w}$ with $w$ representing the portfolio weight vector and Σ the variance–covariance matrix of asset returns.

The primary advantage of the Variance–Covariance method lies in its computational efficiency and analytical simplicity. However, its accuracy critically depends on the validity of the normality assumption and the stability of return volatility. When these assumptions are violated—particularly in markets exhibiting fat-tailed distributions or volatility clustering—the VC method may yield biased risk estimates.

### 2.3. VaR Estimation Using Monte Carlo Simulation

Monte Carlo Simulation (MCS) is a flexible, computation-based approach that estimates VaR by generating a large number of potential future return scenarios through random sampling (Glasserman, 2004). Unlike parametric methods, Monte Carlo simulation can accommodate complex return dynamics, including non-normality, skewness, kurtosis, and nonlinear dependence structures among assets.

In this approach, portfolio returns are simulated $n$ times based on the assumed data-generating process. For each simulated path, the portfolio loss is computed, and VaR is obtained as the empirical α-quantile of the simulated loss distribution. To enhance numerical stability, the VaR estimate is often expressed as the average of VaR values across simulations:

$$VaR_{MC} = \frac{1}{n}\sum_{i=1}^{n} VaR_{(i)} \qquad (4)$$

where $VaR_{(i)}$ denotes the VaR estimate obtained from the $i$-th simulation.

Monte Carlo simulation is particularly well suited for risk estimation in volatile and non-linear markets, as it allows for richer modelling of return distributions and tail behaviour. However, this flexibility comes at the cost of higher computational requirements compared to parametric approaches.

#### 2.3.1. Conditional Value at Risk (CVaR)

Conditional Value at Risk (CVaR), also referred to as Expected Shortfall (ES), is a coherent risk measure developed to address the fundamental limitations of Value at Risk (VaR), particularly its inability to capture tail risk (Rockafellar & Uryasev, 2000). CVaR quantifies the expected loss given that the loss has exceeded the VaR threshold, thereby providing a more comprehensive representation of extreme downside risk (Uryasev, 2000).

Mathematically, for a loss random variable $X$ and a confidence level $(1-\alpha)$, CVaR is defined as the conditional expectation of losses exceeding VaR:

$$CVaR_\alpha(X) = E[X \mid X \geq VaR_\alpha(X)] \tag{5}$$

This definition highlights the ability of CVaR to capture not only the probability but also the magnitude of extreme losses.

#### 2.3.2. CVaR Estimation Using the Variance–Covariance Method

Under the assumption that portfolio returns follow a normal distribution, CVaR can be analytically derived using the Variance–Covariance (VC) approach. In this framework, CVaR is expressed as:

$$CVaR_{VC} = VaR_\alpha + \frac{\sigma_p}{1-\alpha}\phi(Z_\alpha) \tag{6}$$

where $\sigma_p$ denotes the portfolio standard deviation, $Z_\alpha$ is the critical value of the standard normal distribution at confidence level $\alpha$, and $\phi(.)$ represents the probability density function of the standard normal distribution.

This formulation explicitly shows that CVaR exceeds VaR and incorporates the tail thickness of the assumed distribution. While computationally efficient, the accuracy of this approach depends critically on the validity of the normality assumption.

#### 2.3.3. CVaR Estimation Using Monte Carlo Simulation

In the Monte Carlo simulation framework, CVaR is estimated empirically from simulated portfolio loss distributions. After generating a large number of simulated loss outcomes, VaR is first determined as the empirical α-quantile of the loss distribution. CVaR is then computed as the average of losses that exceed the estimated VaR:

$$CVaR_{MC} = \frac{1}{k}\sum_{i=1}^{k} L_i \tag{7}$$

where $k$ denotes the number of simulated loss observations satisfying $L_i \geq VaR_\alpha$ and $L_i$ represents the individual loss values beyond the VaR threshold.

The Monte Carlo approach offers substantial flexibility in modelling complex return dynamics, including non-normal distributions, skewness, and fat tails. Consequently, it is particularly suitable for estimating CVaR in highly volatile or nonlinear market environments, albeit at the cost of increased computational intensity.

### 2.4. Data

The object of this study comprises ten energy sector stocks, focusing on the oil and gas as well as coal subsectors, which were listed on the Indonesia Stock Exchange (IDX) during 2023. The list of selected stocks is presented in Table 1.

**Table 1.** Energy Sector Stocks in the Oil and Gas and Coal Subsectors

| No. | Stock Code | Company Name |
|---|---|---|
| 1 | PTBA | PT Bukit Asam Tbk |
| 2 | MBAP | PT Mitrabara Adiperdana Tbk |
| 3 | MEDC | PT Medco Energi Internasional Tbk |
| 4 | ADRO | PT Adaro Energy Indonesia Tbk |
| 5 | ITMG | PT Indo Tambangraya Megah Tbk. |
| 6 | INDY | PT Indika Energy Tbk |
| 7 | ESSA | PT Surya Esa Perkasa Tbk |
| 8 | BUMI | PT Bumi Resources Tbk |
| 9 | ENRG | PT Energi Mega Persada Tbk |
| 10 | ELSA | PT Elnusa Tbk |

The variables analyzed in this study consist of monthly closing stock prices observed over the period from January 2020 to December 2024. All data processing and analysis were conducted using RStudio, which facilitates the implementation of statistical and computational simulation methods.

## 3. RESEARCH FRAMEWORK

The research framework was developed in a systematic and sequential manner to measure and compare portfolio risk using VaR and CVaR under the Variance–Covariance (VC) and Monte Carlo (MC) Simulation approaches. The research stages are described as follows:

1. Initial Stock Selection

   The study begins with the selection of the most representative stocks from the ten energy sector stocks using the Weighted Scoring Method, based on relevant fundamental indicators and liquidity measures.

2. Stock Return Calculation

   Returns for each selected stock are computed using the simple return formula:

   $$r_{it} = \frac{P_{it} - P_{i,t-1}}{P_{i,t-1}} \quad (8)$$

   where $P_{it}$ denotes the closing price of stock $i$ at time $i$.

3. Estimation of Statistical Parameters

   The mean return vector $\mu$ and the variance–covariance matrix $\Sigma$ are estimated as the basis for portfolio construction.

4. Optimal Portfolio Construction

   The optimal portfolio is constructed using the Maximum Sharpe Ratio (MSR) approach by maximizing the ratio between expected portfolio return and portfolio risk.

5. Portfolio Return and Risk Measurement

   The expected portfolio return is calculated as $\mu_p = w^{\mathrm{T}}\mu$, while portfolio risk is measured using the standard deviation $\sigma_p = \sqrt{w^T \sum w}$

6. Value at Risk (VaR) Estimation

   VaR is estimated using both the Variance–Covariance method and Monte Carlo Simulation at specified confidence levels.

7. Conditional Value at Risk (CVaR) Estimation

   CVaR is computed using the Variance–Covariance (VC) and Monte Carlo (MC) approaches as the average loss exceeding the corresponding VaR threshold.

8. Risk Model Validation

The accuracy of the VaR and CVaR models is evaluated through backtesting procedures, including the application of the Kupiec Test to assess the adequacy of VaR violation frequencies.

9. Comparative Risk Analysis

   In the final stage, the performance of VaR and CVaR obtained from both approaches is compared to evaluate their effectiveness in capturing extreme portfolio risk.

The overall research flow can be visually summarized in the following flowchart diagram.

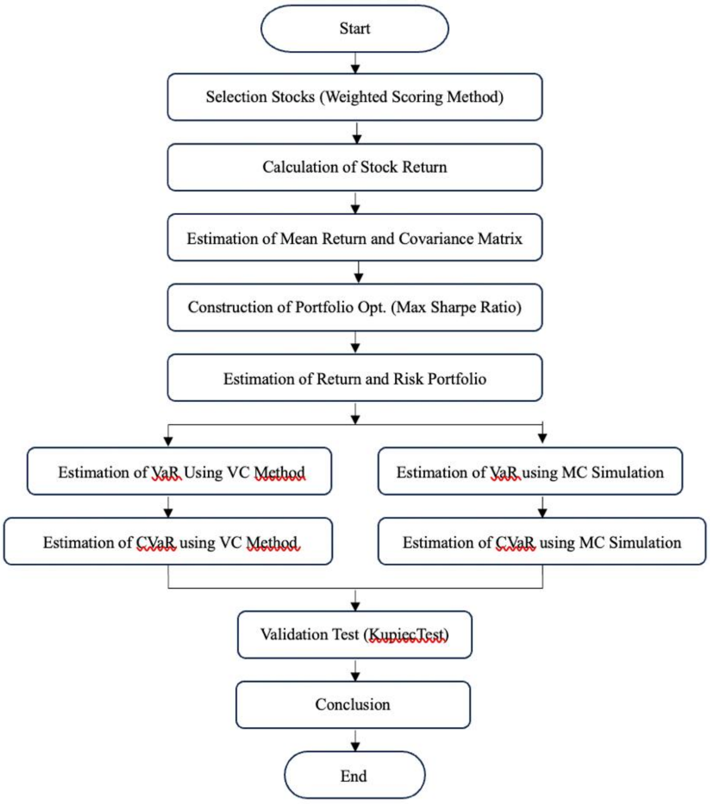


**Figure 1.** Research Flowchart

## 4. RESULTS AND DISCUSSION

### 4.1. Initial Stock Selection

This study focuses on stocks classified in the energy sector. According to data from the Indonesia Stock Exchange (IDX), ten stocks belong to the oil and gas and coal subsectors. Table 2 presents the annual financial data of these stocks, including six indicators: market capitalization, return on equity (ROE), debt-to-equity ratio (DER), net profit margin (NPM), trading volume, and dividend yield.

Table 2 shows that MBAP, PTBA, MEDC, ADRO, ITMG, and INDY demonstrate superior financial performance, characterized by higher ROE and NPM, lower DER, and relatively higher dividend yields. In contrast, ESSA, BUMI, ENRG, and ELSA exhibit higher leverage, weaker profitability, and limited dividend distribution. Overall, firms with strong profitability, low leverage, and consistent dividend payments tend to be medium- to large-cap energy companies with mature operational structures.

**Table 2.** Financial Performance of Ten Stocks in the Oil and Gas and Coal Subsectors

| No. | Stock Code | Market Cap | ROE (%) | DER | NPM (%) | Volume Trading Volume (M) | Dividend Yield (%) |
|---|---|---|---|---|---|---|---|
| 1 | PTBA | 31.8 | 58.2 | 0.32 | 35.8 | 32.1 | 12.5 |
| 2 | MBAP | 9.4 | 64.3 | 0.21 | 39.2 | 1.2 | 15.8 |
| 3 | MEDC | 35.2 | 24.1 | 1.45 | 21.3 | 28.5 | 3.2 |
| 4 | ADRO | 28.5 | 45.7 | 0.45 | 28.4 | 25.3 | 8.7 |
| 5 | ITMG | 21.2 | 42.3 | 0.38 | 25.6 | 18.7 | 9.2 |
| 6 | INDY | 15.7 | 38.9 | 0.85 | 22.3 | 12.4 | 7.5 |

| No. | Stock Code | Market Cap | ROE (%) | DER | NPM (%) | Volume Trading Volume (M) | Dividend Yield (%) |
|---|---|---|---|---|---|---|---|
| 7 | ESSA | 12.4 | 18.2 | 1.89 | 15.7 | 15.2 | 2.8 |
| 8 | BUMI | 8.2 | 15.2 | 2.45 | 12.8 | 45.2 | 0 |
| 9 | ENRG | 8.5 | 12.3 | 2.34 | 11.2 | 8.7 | 0 |
| 10 | ELSA | 2.1 | 8.5 | 1.67 | 7.8 | 5.4 | 1.2 |

For optimal portfolio construction, three stocks were selected using the Weighted Scoring Method (WSM), a multicriteria decision-making framework that aggregates six fundamental indicators into a composite score (Triantaphyllou, 2000; Adiyana, 2022 and Putri et al., 2025). The WSM results indicate that MBAP and PTBA achieved the highest scores (74.82 and 72.95, respectively), followed by MEDC (56.80). WSM is employed as a preliminary screening mechanism to ensure fundamental strength and liquidity prior to subsequent risk evaluation using VaR and CVaR.

### 4.2. Statistics Description

Figure 2 depicts the monthly returns of PTBA, MBAP, and MEDC over the period January 2020 – December 2024. Overall, the three stocks exhibit fluctuating return patterns with heterogeneous levels of volatility. MEDC displays relatively higher volatility, as indicated by larger return fluctuations, whereas PTBA and MBAP show comparatively more stable return dynamics, despite exhibiting significant temporal variation.

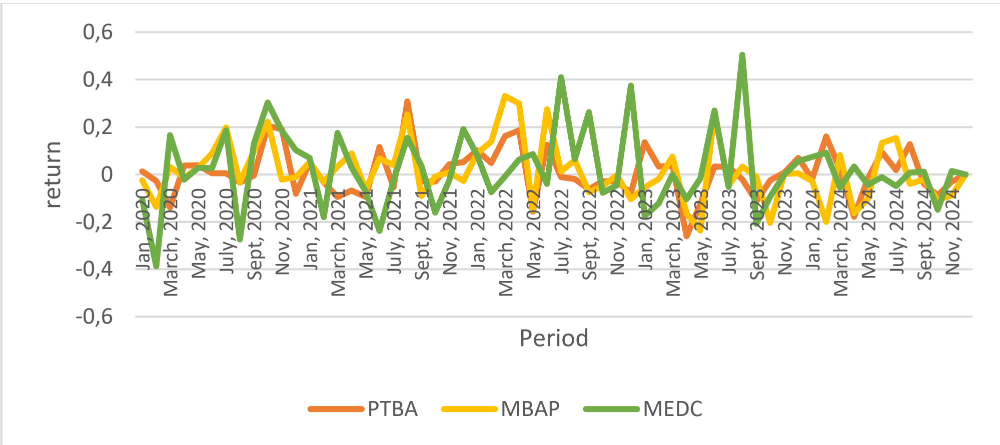


**Figure 2**. Monthly return movements of PTBA, MBAP, and MEDC during the period January 2020 – December 2024.

Meanwhile, Table 3 reports the expected returns and return standard deviations of the three selected stocks, namely PTBA, MBAP, and MEDC. The results indicate that MEDC yields the highest expected return (2.27%), followed by MBAP (1.36%) and PTBA (0.77%). However, MEDC is also associated with the highest risk, as reflected by its return standard deviation of 0.1618. In contrast, PTBA exhibits the lowest expected return but also the lowest volatility (0.1004), suggesting greater return stability. MBAP occupies an intermediate position with an expected return of 1.36% and a standard deviation of 0.1245.

**Table 3.** The Expected Return and Standard Deviation of Three Stocks

| No. | Stock Code | Expected Return | Standard Deviation Return |
|---|---|---|---|
| 1 | PTBA | 0.0077 | 0.1003 |
| 2 | MBAP | 0.0136 | 0.1245 |
| 3 | MEDC | 0.0227 | 0.1618 |

### 4.3. Maximum Sharpe Ratio Portfolio

The mean–variance portfolio optimization framework is built upon the assumption that asset returns are normally distributed or, equivalently, that investor preferences can be fully characterized by the first two moments of the return distribution, namely the mean and variance. Therefore, validating the normality assumption of asset returns is a crucial preliminary step to ensure the appropriateness and reliability of the mean–variance–based optimal portfolio construction employed in this study.

In this study, normality is assessed using the Shapiro–Wilk test and visual inspection through Q–Q plots for three stocks: PTBA, MBAP, and MEDC. Table 4 presents the results of the normality test using the Shapiro–Wilk test. The Shapiro–Wilk test results indicate that all stocks have p-values greater than 0.05, namely PTBA (p-value = 0.2980), MBAP (p-value = 0.0630), and MEDC (p-value = 0.1443). These results provide no statistical evidence to reject the null hypothesis of normality, suggesting that the return distributions of all three stocks are consistent with a normal distribution.

**Table 4.** The Shapiro–Wilk Normality Test Results

| Stock Code | P-value | Interpretation |
|---|---|---|
| PTBA | 0.2980 | Normal Distribution |
| MBAP | 0.0630 | Normal Distribution |
| MEDC | 0.1443 | Normal Distribution |

These findings are further supported by the Q–Q plots (see Figure 3), which show that the data points closely follow the diagonal reference line without significant deviations, indicating a good fit to the normal distribution.

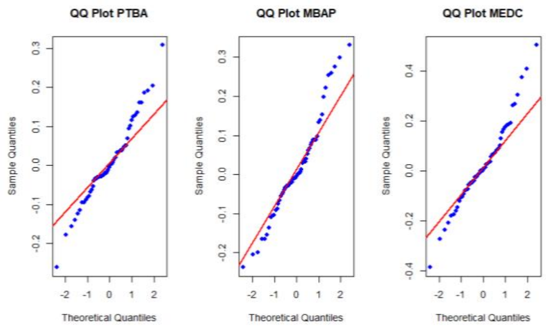


**Figure 3.** Normality Test Using Q–Q Plot.

The optimal portfolio is constructed using the Maximum Sharpe Ratio (MSR) approach, which seeks to maximize the ratio of excess expected return over the risk-free rate to portfolio risk, as measured by the standard deviation. The optimization problem formulated in Equation (1) is solved using quadratic programming, which provides an efficient numerical solution for constrained portfolio optimization problems.

Specifically, the optimization is implemented using the solve.QP function available in R Studio, which is commonly applied in empirical portfolio optimization studies (Gerrad & Johnson, 2015; Saber & Sulaiman, 2022). After performing the optimization procedure, the optimal Maximum Sharpe Ratio portfolio weights for the selected stocks are obtained and presented in Table 5.

**Table 5.** The Optimal Portfolio Weight

| No. | Stock Code | Weight | Percentage |
|---|---|---|---|
| 1 | PTBA | 0.6301 | 63.01% |
| 2 | MBAP | 0.1394 | 13.94% |
| 3 | MEDC | 0.2304 | 23.05% |

The optimal portfolio constructed using the Maximum Sharpe Ratio (MSR) approach yields asset weights of 63.01% for PTBA, 23.05% for MEDC, and 13.94% for MBAP. Based on this allocation, the optimal portfolio generates an expected return $\mu_p$ of 1.2027% with a risk level, measured by the standard deviation $\sigma_p$, of 8.0811%.

Risk measures based solely on standard deviation capture total volatility but may fail to adequately reflect extreme downside risk, particularly in the energy sector, which is characterized by high volatility and fat-tailed return distributions (Takaishi, 2023; Widiastuti et al., 2025). Therefore, further evaluation using tail-risk-based measures is essential to assess the portfolio's resilience under extreme market conditions. The application of VaR and CVaR to the MSR portfolio enables a more comprehensive assessment of downside risk.

### 4.4. VaR and CVaR Using the Variance–Covariance Method

The estimation of CVaR using the variance–covariance method begins with the computation of asset returns and the estimation of distribution parameters, namely the mean return $\mu$ and the variance–covariance matrix $\Sigma$. Given the portfolio weight vector $w$, portfolio risk is represented by the portfolio standard deviation $\sigma_p$, under the assumption that portfolio returns follow a normal distribution. At this stage, VaR at a specified confidence level is first calculated as the threshold for extreme losses. Subsequently, CVaR is computed as the expected portfolio loss conditional on losses exceeding the VaR level.

**Table 6.** VaR and CVaR Estimates Using the Variance–Covariance Method

| Significance Level | VaR | CVaR |
|---|---|---|
| 95% | 13.2738 | 16.9515 |
| 99% | 19.2718 | 22.2542 |

Table 6 shows that both VaR and CVaR increase as the confidence level rises from 95% to 99%, reflecting a more conservative assessment of portfolio risk. At the 95% confidence level, the VaR of 13.2738 represents the estimated maximum portfolio loss under normal market conditions, whereas at the 99% confidence level, VaR increases to 19.2718, indicating substantially higher potential losses under extreme market scenarios.

CVaR values consistently exceed VaR, amounting to 16.9515 at the 95% confidence level and 22.2542 at the 99% confidence level. This confirms that CVaR captures the expected loss conditional on losses exceeding the VaR threshold. The widening gap between VaR and CVaR, particularly at the 99% confidence level, highlights the presence of tail risk that is not fully captured by VaR. These findings underscore the superior ability of CVaR to provide a more comprehensive measure of extreme downside risk.

### 4.5. VaR and CVaR Using the Monte Carlo Simulation

The procedure for estimating VaR and CVaR using the Monte Carlo Simulation method begins with modeling portfolio returns as normally distributed, with parameters $\mu_p$= 1,2027% and $\sigma_p$= 8,0811%. Based on these parameters, 50,000 simulation are generated to obtain an empirical distribution of portfolio returns that more flexibly captures risk behaviour.

The use of 50,000 simulation trials is intended to enhance the accuracy of risk estimation, as a larger number of scenarios allows for a more realistic assessment of market uncertainty. The next step involves determining VaR at the 95% and 99% confidence levels as the corresponding quantiles of the simulated return distribution. CVaR is then computed as the expected portfolio loss conditional on returns falling below the VaR threshold, by averaging all simulated losses that exceed the VaR level.

**Table 7.** VaR and CVaR Estimates Using the Monte Carlo Simulation

| Significance Level | VaR | CVaR |
|---|---|---|
| 95% | 13.2923 | 16.9095 |
| 99% | 19.1942 | 21.9609 |

Table 7 reports the Monte Carlo–based VaR and CVaR estimates, revealing a consistent increase in risk as the confidence level rises. At the 95% confidence level, the VaR value of 13.2923 indicates that under normal market conditions, portfolio losses are not expected to exceed this level with 95% probability. However, the corresponding CVaR value of 16.9095 indicates that, when losses exceed the VaR threshold, the expected loss becomes substantially larger. This highlights the ability of CVaR to provide additional information on extreme downside risk that is not captured by VaR.

At the 99% confidence level, VaR increases to 19.1942, reflecting higher potential losses under highly adverse market scenarios. Consistently, CVaR also rises to 21.9609, indicating that the expected loss in the worst-case scenarios is considerably greater than the VaR limit. The persistent gap between VaR and CVaR at both confidence levels confirms that CVaR is more conservative and informative in capturing tail risk. These findings support the effectiveness of the Monte Carlo Simulation approach in modeling extreme portfolio risk and demonstrate the superiority of CVaR over VaR as a risk measure.

### 4.6. Validation Test

The validation of the VaR model is conducted using a backtesting approach with the Kupiec test. Backtesting refers to the process of evaluating the performance of a model or strategy by comparing its estimated or predicted values with realized historical data (Kupiec, 1995; Rosyidah, 2024). VaR is estimated out-of-sample using a rolling-window approach, in which model parameters are calibrated based on historical data within each window and then used to forecast VaR for the subsequent period.

The backtesting procedure is implemented as follows. First, the length of the estimation window is specified; in this study, a window of 30 periods (December 2020–June 2022) is employed. Second, VaR is estimated using the initial window. Third, the estimation window is shifted forward by one period, and the

VaR estimation is repeated iteratively until the end of the sample, thereby generating a series of out-of-sample VaR estimates.

Next, VaR exceedances (violations) are identified according to the indicator function:

$$I_t = \begin{cases} 1; & r_t < -VaR_\alpha \\ 0; & \text{otherwise} \end{cases} \quad (9)$$

Where $r_t$ denotes the realized portfolio return at time $t$.

The empirical violation rate $p$ is then computed as the ratio of the total number of exceedances $N$ to the total number of observations $T$. Finally, the validity of the VaR model is evaluated using the Kupiec Proportion of Failures (POF) test, with the likelihood ratio statistic given by

$$LR_{up} = -2ln\left[\frac{(1-\alpha)^{T-N}\ \alpha^N}{(1-p)^{T-N}\ p^N}\right] \quad (10)$$

In the context of CVaR, the Kupiec test is not directly applicable, as CVaR is not a quantile-based measure but represents the expected loss in the tail of the return distribution. Nevertheless, the validity of CVaR is commonly assessed indirectly through the validity of VaR as its defining threshold. That is, if the VaR estimate is validated by the Kupiec test, the CVaR estimate—computed from observations exceeding the VaR level—can be regarded as methodologically consistent. Accordingly, empirical studies often report the Kupiec test as the primary validation tool for VaR models, while CVaR evaluation relies on analyses of stability, conservativeness, and comparative tail loss behavior (Rockafellar & Uryasev, 2000; McNeil et al., 2015; Vaniya & Gor, 2022).

Based on the Kupiec test results, a p-value of 0.6568 is obtained, which exceeds the significance level α=0.05; therefore, the null hypothesis is not rejected. This indicates that the VaR estimates are statistically valid and adequately represent the portfolio's market risk. Since CVaR is derived from losses exceeding the VaR threshold, the validation of VaR also supports the consistency and reliability of the CVaR estimates. Consequently, both risk measures can be used complementarily to assess portfolio risk, particularly in capturing extreme downside losses.

## 5. CONCLUSIONS AND RECOMMENDATIONS

### 5.1. Conclusion

This study analyzes the risk of energy sector stock portfolios listed on the Indonesia Stock Exchange using Value at Risk (VaR) and Conditional Value at Risk (CVaR) estimated through the Variance–Covariance and Monte Carlo Simulation methods. The optimal portfolio constructed based on the Maximum Sharpe Ratio criterion consists of PTBA (63.01%), MEDC (23.05%), and MBAP (13.94%). This allocation yields an expected return of 1.2027% with a risk level, measured by the standard deviation, of 8.0811%, indicating a favorable trade-off between return and risk.

Risk estimation results at the 95% and 99% confidence levels indicate that CVaR consistently produces higher risk estimates than VaR, highlighting its superiority in capturing extreme downside risk (tail risk) that cannot be adequately identified by VaR. A comparative analysis of the estimation methods shows that the Variance–Covariance approach yields slightly higher CVaR estimates than the Monte Carlo method, indicating a more conservative risk assessment. This finding can be attributed to the fulfillment of the normality assumption for portfolio returns, which is a key prerequisite of the Variance–Covariance approach. Under conditions where return distributions are approximately normal and market conditions are relatively stable, the Variance–Covariance method provides efficient and reliable risk estimates.

Furthermore, backtesting results using the Kupiec test confirm that the VaR model satisfies statistical validity criteria, suggesting that the estimated risk measures are reliable for market risk assessment. Overall, the findings emphasize that the choice of risk measurement method should account for the underlying return distribution characteristics: the Variance–Covariance method is more appropriate when the normality assumption holds, whereas the Monte Carlo Simulation method offers greater flexibility under volatile and non-normal market conditions.

### 5.2. Recommendations

Based on the findings of this study, future research is encouraged to extend risk measurement approaches by incorporating non-normal return distribution models, such as the Student's t-distribution or Extreme Value Theory (EVT), to better accommodate fat-tailed and skewed return characteristics commonly observed in turbulent financial markets. This extension is particularly important when the normality assumption underlying the Variance–Covariance method is violated, potentially leading to biased risk estimates. In addition, expanding the scope of analysis to other sectors or employing data across crisis and non-crisis periods would provide a more comprehensive understanding of the robustness and reliability of different portfolio risk measurement methods.

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